

Mental Math Class Handout

Version 04/21/06 © 2006 Ron Doerfler (www.myreckonings.com)

** Numbers are not just collections of digits; they have properties, such as being near a number.

** PLEASE....Work LEFT-to-RIGHT!

** Try to use EASY numbers near the real ones, then fix it up at the end.

** Have confidence. Don't limit yourself by saying it's "too hard." Don't worry about speed.

ADDITION and SUBTRACTION:

1) Work LEFT-to-RIGHT. Keep a "running total" in your head.

$$245 + 487 = 600 + 120 + 12 = 732$$

2) When subtracting, it is often easier to subtract a larger round number and add the rest back.

$$236 - 185 = 236 - 200 + 15 = 51$$

MULTIPLICATION:

"Break-It-Up" Method: Easiest to understand but the toughest, so try other shortcut methods first.

$$7 \times 35 = 7 \times 30 + 7 \times 5 = 245$$

$$6 \times 24 = 6 \times 20 + 6 \times 4 = 144$$

$$24 \times 32 = 20 \times 32 + 4 \times 32 = 768$$

"Good Neighbor" Method: Use a nice neighboring number to multiply by, then fix it up at the end.

$$9 \times 44 = 10 \times 44 - 44 = 396$$

$$11 \times 44 = 10 \times 44 + 44 = 484$$

$$19 \times 34 = 20 \times 34 - 34 = 646$$

$$41 \times 56 = 40 \times 56 + 56 = 2296$$

Useful "Trick": Squaring a 2-digit number near 50

Find how far the 2-digit number is from 50. Add it to 25. Square it and put it at the end.

54 x 54: 54 is 4 more than 50, so $(25 + 4) = 29$ End with $4 \times 4 = 16$, so we get 2916

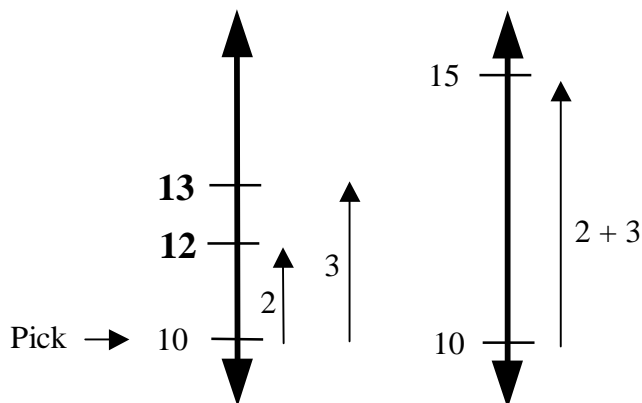
43 x 43: 43 is 7 less than 50, so $(25 - 7) = 18$ End with $7 \times 7 = 49$, so we get 1849

Multiplying by 25, 50, or 75

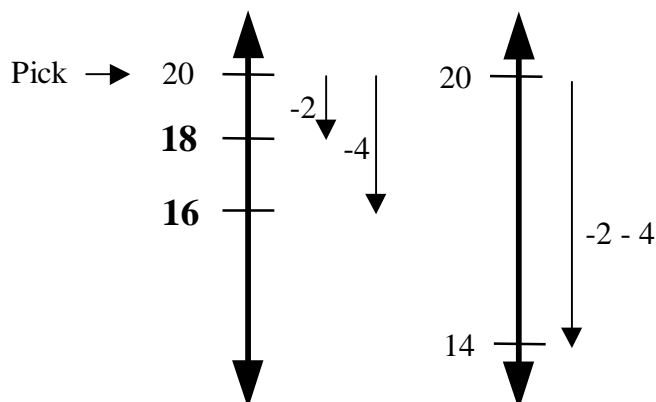
50 is the same as $100/2$, so multiply by 100 and divide by 2: $34 \times 50 = 3400 / 2 = 1700$
You can also remember that $25 = 100/4$ and $75 = 300/4$.

“Lotto” Method: Our really powerful method for close numbers (say, within 15 or 20)

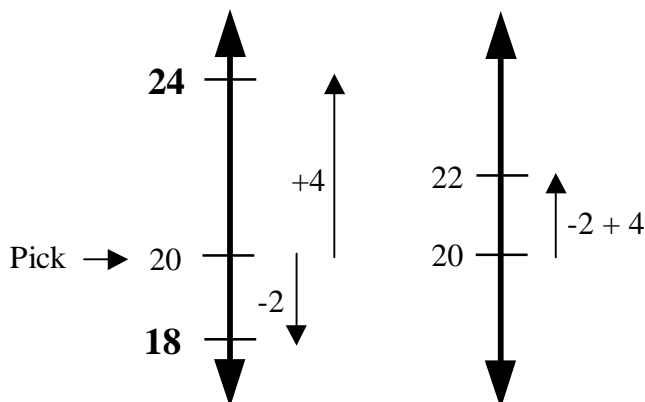
Pick a nearby number that is easy to multiply by and use this as a “Base” for the calculation. To find the other number, add the differences from the original numbers. Multiply the differences and add.



$$12 \times 13 = 10 \times 15 + 2 \times 3 = 156$$



$$18 \times 16 = 20 \times 14 + 2 \times 4 = 288$$



$$18 \times 24 = 20 \times 22 - 2 \times 4 = 432$$

Pick a Lower Number:

$$13 \times 17 \text{ Pick } 10, \text{ then } 10 \times 20 + 3 \times 7 = 221$$

$$35 \times 35 \text{ Pick } 30, \text{ then } 30 \times 40 + 25 = 1225$$

$$23 \times 32 \text{ Pick } 20, \text{ then } 20 \times 35 + 3 \times 12 = 736$$

Pick a Higher Number:

$$18 \times 19 \text{ Pick } 20: 20 \times 17 + 1 \times 2 = 342$$

$$35 \times 38 \text{ Pick } 40: 40 \times 32 + 5 \times 3 = 1295$$

An old way to avoid learning tables past 5x5:

$$\begin{array}{r} 8 \times 7 \\ 3 \times 2 \\ \hline 56 \end{array}$$

Find 8×7 :

3 and 2 are $(10-7)$ and $(10-8)$.

Then: 5 is $(7-2)$ or $(8-3)$
and 6 is (3×2)

Answer: 56

Pick a Between number : (Subtract, Don't Add!)

$$27 \times 35 \text{ Pick } 30: 30 \times 32 - 3 \times 5 = 945$$

$$17 \times 26 \text{ Pick } 20: 20 \times 23 - 3 \times 6 = 442$$

Additional Material for Classes Familiar with Algebraic Notation

Multiplying two numbers far apart:

The biggest problem can be getting the two numbers close enough to each other to use the Lotto Method. Let's say you have $32 \times 72 = ?$. You can:

- 1) Subtract the larger number from 100:

$$32 \times 72 = 32 \times (100 - 72) = 3200 - 32 \times 28$$

- 2) Divide the larger number by 2 and then multiply it back at the end:

$$32 \times 72 = 2 \times (32 \times 36)$$

- 3) Break out 50 first, which is easy to multiply by since it equals 100/2:

$$32 \times 72 = 32 \times 50 + 32 \times 22 = 3200/2 + 32 \times 22 = 1600 + 32 \times 22$$

Multiplying two numbers ending in 5:

As seen in the example of 35×35 under the Lotto section in which a Lower number is picked, the square of a number ending in 5 (say, $a5$) is given by $a(a+1)$ with 25 appended:

$$\text{From Lotto section: } 35 \times 35 \text{ Pick } 30, \text{ then } 30 \times 40 + 25 = 1225$$

$$\text{or this way: } 35 \times 35 = (3 \times 4) \mid 25 = 1225$$

This is a special case of a more general method for multiplying two different numbers ending in 5:

$$a5 \times b5 = [a \times b + (a+b)/2] \mid 25$$

The third term in the brackets is the average of **a** and **b**. If the average ends in .5, drop the .5 but append 75 instead of 25:

$$35 \times 55 = (3 \times 5 + 4) \mid 25 = 1925$$

$$55 \times 85 = (5 \times 8 + 6) \mid 75 = 4675$$

$$35 \times 35 = (3 \times 3 + 3) \mid 25 = (3 \times 4) \mid 25 = 1225$$

Squaring a number one away from a round number:

$$a^2 = (a-1)^2 + (a-1) + a \quad \text{so: } 31^2 = 30^2 + 30 + 31 = 961$$

$$a^2 = (a+1)^2 - (a+1) - a \quad \text{so: } 29^2 = 30^2 - 30 - 29 = 841$$

Squaring a number ending in one:

$a1^2 = a^2 \parallel (2 \times a) \parallel 1$ where the middle section is one digit (add any upper digit to left section).

$$21^2 = 4 \parallel 4 \parallel 1 = 441$$

$$71^2 = 49 \parallel 14 \parallel 1 = 5041$$