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# Edward's Challenge

A Mathematical Puzzle by Ron Doerfler

<http://www.myreckonings.com>

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*Y*OU WON'T GET THIS ONE, MY FRIEND!"

The large, burly man, disregarding the sidelong glances of others in the various settings of the room, threw himself into a leather chair. "This is the best I've got."

No salutations were necessary, of course. Edward and I were old friends, two individuals with a certain bent for one-upmanship and a particular fascination for puzzles. For years now, we had met monthly in what we used to call a smoking room, now (due to certain rules changes) simply a meeting room. We had mixed success with each other's challenges, due not to any lack of talent but rather to the ferocity with which we played.

"Alright," said Edward after settling in, "You know that when we divide a number  $N$  by another number, we can find the remainder in certain cases by working simple arithmetic on digits or groups of digits in  $N$ . For example, if we want to find whether  $N$  is divisible by 9, we can simply add the digits of  $N$ , and then add the digits of this new number, and so forth, and all these new numbers will have the same remainder as  $N$  will have when divided by 9. This is the digit-sum used in "casting out nines" to check arithmetic."

This was unlike Edward. I began to feel uncomfortable.

"Well, there are others," he continued. "I have in mind a number  $N$ , or a collection of numbers  $N$ , and from information I give on such remainders, you will be asked to provide certain information about the digits of  $N$ ."

So this was it.

"We will concern ourselves with a positive integer  $N$ , a number containing sixteen digits."

"Good Lord!"

"Well, yes, it's enormous, I suppose, but I have to circumvent the electronic nuisances of the day. This number  $N$  has useful properties, however, to reason about. All digits are even (0,2,4,6, or 8), and leading zeros are allowed. In general, all digits in even positions, or places, in the leftmost, upper half of  $N$  also appear in even places in the lower half, although they can be mixed up in these places. The same is true of the digits in the odd positions. There are three exceptions, however. In the lower half of  $N$ , one even-place digit and two odd-place digits are each increased by 2 over their counterparts in the upper half. Let me provide an example."

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An example—this was unheard of. Edward wrote the eight digits 08424680 on a sheet of paper and appended a mix of the same digits, retaining even/odd places within the number, arriving at 0842468086400248. He then chose two odd-place digits (2 and 6) and one even-place digit (0) to increase by 2 in the lower half, yielding

0842468088402448

“That,” said Edward, “is a valid N.”

I mentally calculated the number of days in the next month. This looked like it might take some time.

“Don’t worry, it’s not that bad. Now I have my first two questions:

1. What is the minimum value that N can be with the information given up to now?
2. What is the maximum value that N can be with the information given up to now?”

“Now I can tell you that when N is represented by a 17-digit octal number (base 8), the sum of the odd-place “digits” minus the even-place “digits” leaves a remainder of 7 when divided by one more than the base, i.e., by 9 decimal. When N is represented as a 15-digit duodecimal number (base 12), the sum of all the “digits” is evenly divisible by one less than the base, or 11 decimal.”

“Ah,” I interjected. “The digit sum in any base reveals divisibility by one less than the base, and the odd-place digit sum minus the even-place digit sum reveals divisibility by one more than the base. You have told me that, in decimal, when N is divided by 9, a remainder of 7 occurs, and also that N is evenly divisible by 11!”

Edward looked annoyed. Without replying, he continued, “Now here are the rest of the questions:

3. What possible value(s) can the sum of the odd-place digits of N be?
4. What possible value(s) can the sum of the even-place digits of N be?
5. What is the minimum value that N can be?
6. What is the maximum value that N can be?”

“That’s it,” said Edward.

I was somewhat surprised—I expected more information after all the build-up. There seemed no doubt that every bit of data given was important. My question to you is:

“Can you help?”

\*\* The solution to this puzzle can be found at  
[http://www.myreckonings.com/Dead\\_Reckoning/Online/EdwardsChallengeSolution.pdf](http://www.myreckonings.com/Dead_Reckoning/Online/EdwardsChallengeSolution.pdf)