
The Sensational Mentalist

A Mathematical Puzzle by Ron Doerfler

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Author's Note:

Finding the remainder when a number is divided by a smaller one is useful for determining factors of a number and for checking results of calculations. This task is much easier when dividing by certain numbers. For example, the nines-test (also called the digit-sum test or "casting out nines") is a common error-checking mechanism. We can simply add the digits of a number to get a much smaller number, and its remainder after dividing by 9 is the same as for the original number. There are other cases as well, e.g., we can add pairs of digits to get a smaller number whose remainder after dividing by 99 is the same as for the original number, and we can subtract the sum of the even digits from the sum of the odd digits to get a smaller number whose remainder after dividing by 11 is the same as for the original number.

To make a puzzle, we will do the reverse; we will be given the remainder when a number N is divided by another number, and our task will be to extract information about the patterns of digits in N . Only basic mathematics is used in solving the puzzle, and pencil and paper is all that is needed.

CHARLES THE SENSATIONAL cleared his throat as he asked for a volunteer from the audience. Charles, a dependable but middling mentalist (his friends referred to him as a medium), graciously welcomed the nervous participant onto the stage. He thought the man said his name was Jim, but he wasn't sure he remembered correctly—no matter.

"Now, I would like you to secretly construct a number N ," spoke Charles, handing the man a notepad. "It must be 12 digits long. You can choose the digits, and you can repeat digits, subject to the following constraint: any digit in an even place in the leftmost 6 digits of the number must also occur in an even place in the rightmost 6 digits of the number, although they can be located in different even places in the two halves. Also, any digit in an odd place in the leftmost 6 digits of the number must also occur in an odd place in the rightmost 6 digits of the number, although they can be located in different odd places in the two halves."

The man looked confused. Charles decided to drop the formalities.

"Look, uh, sir, just choose any 6-digit number and write it down. Now take the digits of this number and mix them up, if you wish, but make sure that the digits that were in the even places are still in even places. Now append this last 6-digit number to the original 6-digit number, and you create this 12-digit number N . For example, good numbers are 123456361452 or 002234203204 and so forth. There are millions of numbers between 000000000000 and 999999999999 that fit the bill."

“Jim” seemed satisfied and discreetly wrote a number N on his notepad, hiding it from Charles and the audience.

“Now, sir, divide this number by 99 and tell me the remainder, if any,” spoke Charles, back in stride. Shortly thereafter, having received a reply of “79” from the man, Charles proudly announced, “In the parlance of modular arithmetic, we would say that $N \bmod 99$ equals 79.” The audience oohed.

Charles plucked a sheet of paper and began scribbling furiously (hence the poor reputation). A minute later, he straightened up and addressed the audience. “With my mental powers, I now have gleaned incredible knowledge about the number N that was chosen!” He then proceeded to proclaim the following information, which is asked of you:

1. The maximum number that N can be.
2. The minimum number that N can be.
3. The list of possible sums of all the digits of N (a surprisingly short list).
4. The list of possible sums of all the digits in the odd places of N .
5. The list of possible sums of all the digits in the even places of N .

The audience wanted more. Charles decided to take a chance. He looked at the man and said, “I will now ask you a series of questions until I deduce the actual number. First, write down on a separate sheet the locations in N of the lowest digit.”

Charles scanned the paper and announced the result—the lowest digit was in the 12th and 4th positions from the right (if this lowest digit were x , N would look like $x \cdots \cdots x \cdots$).

“Next, write down the actual sum of all the digits in the even places of N .” When Charles saw the result, he looked up, somewhat surprised. Without announcing the sum, he began, “Ladies and gentlemen, I now know the value of N ! It is ...”

But before he could continue, a large man in the audience, Jonathon the Tremendous (a great mentalist prone to thoughtless acts), leapt to his feet and shouted, “I know the value of N !”

6. What is the value of N ?

** The solution to this puzzle can be found at
http://www.myreckonings.com/Dead_Reckoning/Online/SensationalMentalistSolution.pdf