Proportional Nomograms

4 Variable Proportion \[ \frac{f_1(u)}{f_2(v)} = \frac{f_3(w)}{f_4(t)} \]

Proportional Design:

Ideal Gas Law: \( pV = nRT \)

Law of Sines: \( \frac{A}{\sin \alpha} = \frac{B}{\sin \beta} \)

\[ \frac{m_1}{m_2} = \frac{m_3}{m_4} \]

True for all types shown here
<table>
<thead>
<tr>
<th>Sunday</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td></td>
<td>11</td>
</tr>
<tr>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
</tr>
<tr>
<td>20</td>
<td>21</td>
<td>22</td>
<td>23</td>
<td>24</td>
<td>25</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>25</td>
</tr>
</tbody>
</table>

**Father's Day**
<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
<td>21</td>
</tr>
<tr>
<td>22</td>
<td>23</td>
<td>24</td>
<td>25</td>
<td>26</td>
<td>27</td>
<td>28</td>
</tr>
<tr>
<td>29</td>
<td>30</td>
<td>31</td>
<td>31</td>
<td>31</td>
<td>31</td>
<td>31</td>
</tr>
</tbody>
</table>

May

Mother's Day

Memorial Day
Equations of more than three variables can be graphically computed using compound nomograms sharing scales. The k-scale is not labeled with scale values. It is called a pivot line. Since the angle A between the scales is 60°, the scales are identical.

\[
\begin{align*}
\gamma &= (n)\gamma - (m)\gamma \\
\gamma &= (a)\gamma + (n)\gamma \\
(m)\gamma - (m)\gamma &= (a)\gamma + (n)\gamma
\end{align*}
\]
Two Classic Nomogram Designs

**Division**
\[ f_3(w) = \frac{f_1(u)}{f_2(v)} \]

Here we have \[ r^2 = \frac{V}{\pi h} \]

**Harmonic Relation**
\[ \frac{1}{f_1(u)} + \frac{1}{f_2(v)} = \frac{1}{f_3(w)} \]

Design:
\[ m_1 = m_2 = \frac{m_3}{2 \cos A} \]

where \( A \) is the angle between each of the 3 scales.
If \( A = 60^\circ \) as below, then \( m_1 = m_2 = m_3 \).

**An “N” or “Z” Chart**

\[ Z = \frac{L f_3(w)}{m_2/m_1 + f_3(w)} \]

**A Concurrent-Scale Nomogram**

The diagonal scale can be floating segment, thus appearing “rather more spectacular” to the casual observer [Douglass 1947].

Standard resistor values can be marked so a convenient combination can be found by playing with the straightedge.
<table>
<thead>
<tr>
<th>Sunday</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Independence Day</td>
<td>Independence Day (Obs.)</td>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td></td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td></td>
<td>19</td>
<td>20</td>
<td>21</td>
<td>22</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td></td>
<td>24</td>
<td>25</td>
<td>26</td>
<td>27</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td></td>
<td>29</td>
<td>30</td>
<td>31</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
April

Easter Sunday

Sunday Monday Tuesday Wednesday Thursday Friday Saturday
Finding real roots graphically


gives \( z = 0.96, 0.252, 0.14 \) gives \( z = 0.47 \), while \( p = 0.15 \) and \( q = 0.11 \) with \( p \). For example, \( p = 0.6 \) and \( q = 0.4 \) triangular region, on its boundary, or


No need to graduate the circle! Read the roots as \(-B\) on lines from \( O \) through the marked location on the circle (here -3.06 for \( A = -0.1, B = 4 \)).


For a real root \( w_1 \) found here, the second root = \( u + w_1 \).


The second root = \( u + w_1 \) for a real root \( w_1 \) found here.


Finding real roots graphically


Linear scales and easy custom ranges for \( a \) and \( b \) for \( Ax^2 + x + B = 0 \), \( (A=0.1, B=4) \) gives \( z = 0.96 \). For example, \( p = 0.6 \) on the circle marked location through the line from \( O \) roots as \( B \) on the circle. Read the gradient the graph at the linear scale.


Their isopleth cuts the circle, two roots are found if the isopleth cuts the circle, no root.


Linear scales and


Graphical Computation


The area of


Solving Polynomial Equations
Nomography was invented in 1880 by Maurice d'Ocagne and was used extensively for many years to provide engineers with fast graphical calculations of complicated formulas to a practical precision.

Nomograms solve equations in 3 or more variables, providing lightning fast, easy calculations to an engineering precision in a form that is easy to reproduce on a photocopier.

Example: $N = (1.2D + 0.47)^{0.68}(0.91T)^{3/2}$

A parallel-scale nomogram

A straightedge (such as the edge of a sheet of paper or a string) called an isopleth is used to connect known values to find the unknown value.

The simplicity of a nomogram can be startling!

Parallel-Scale Design: $f_1(u) + f_2(v) = f_3(w)$

Take logarithms to convert the equation to a sum

$0.68 \log(1.2D + 0.47) + 1.5 \log T = \log N - 1.5 \log 0.91$
Once invented, nomograms were soon applied to time-consuming and repetitive calculations in celestial mechanics. This is an example of a nomogram solving for a variable \( \phi \) that cannot be isolated algebraically.

Kepler's Equation for the relation between the polar angle \( \phi \) of a celestial body in an eccentric orbit and the time elapsed from an initial point:

\[
nt = \phi - \varepsilon \sin \phi
\]

Celestial Parallax: the difference between topocentric and geocentric location when observing comets and minor planets. Done with parallax correction, generally to two digits and in great number to define the orbits:

\[
\Delta p = \text{parallax factor}
\]

\[
\pi s = \text{mean equatorial horizontal parallax of the sun in seconds}
\]

\[
\rho = \text{Earth radius to observation point in term of equatorial radius}
\]

\[
\phi = \text{geocentric latitude of observer}
\]

\[
\delta, H = \text{declination and hour angle of body}
\]

Spherical Triangle relation between declination, latitude, hour angle and azimuth:

\[
\cos \phi = \cos \delta \cos \lambda
\]

\[\text{Celestial Parallax: the difference between topocentric and geocentric location when observing comets and minor planets. Done with parallax correction, generally to two digits and in great number to define the orbits.}\]

\[\text{Spherical Triangle relation between declination, latitude, altitude and azimuth.}\]
Dygograms

The Scottish mathematician and lawyer Archibald Smith first published in 1843 his equations for the magnetic deviation of a ship, or in other words, the error in the ship's compasses from permanent and Earth-induced magnetic fields in the iron of the ship itself. This effect had been noticed in mostly wooden ships for centuries, and broad attempts to minimize it were implemented. But the advent of ships with iron hulls and steam engines in the early 1800s created a real crisis. A mathematical formulation of the deviation for all compass courses for a location at sea was needed in order to understand and compensate for it. Smith invented graphical methods for quickly calculating the magnetic deviation for any ship's course once ship parameters were found, geometric constructions called dynamogoniograms (force-angle diagrams), or dygograms for short.

To construct a dygogram, find the North (N) position by laying out from O the lengths A, B, C, D and E as shown. Draw a circle centered at A and passing through D. The magnetic deviation δ for a magnetic course ζ of North (0°) is the angle XON read on the protractor.

Now extend ND the same distance beyond D to find the South (S) point. The point Q is the intersection with the circle. Continue to create the Limaçon of Pascal figure by moving the midpoint of the segment NS along the circle and marking the endpoints.

δ = A + B sin ζ' + C cos ζ' + D sin 2 ζ' + E cos ζ'

δ is the magnetic deviation (compass correction)
ζ' is the ship compass reading (compass course)
A, D, E, λ, c, f, P and Q are magnetic parameters measured for the ship
H and θ are the horizontal component and dip angle of the Earth's magnetic variation at the ship location
A = arcsin A
B = arcsin [B / (1 + ½ sin D)]
C = arcsin [C / (1 - ½ sin D)]
D = arcsin D
E = arcsin E
A, D, E = constants for ship
B = (1/λ) (c tan θ + P / H)
C = (1/λ) (f tan θ + Q / H)

For any ship compass reading (the compass course ζ'), draw a line from Q at this angle from QN (the red arrow here) and mark the point where it crosses the vertical line OX. Then with dividers construct an arc that passes through O, Q, and this point (the blue circle) and mark a new point where it crosses the limaçon (the magnetic course ζ). The magnetic deviation δ is the angle between OX and this new point as read on the protractor.
<table>
<thead>
<tr>
<th>Sunday</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>6</td>
<td>7  ●</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>Labor Day</td>
<td></td>
<td></td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
</tr>
<tr>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
</tr>
<tr>
<td>19</td>
<td>20</td>
<td>21</td>
<td>22  ○</td>
<td>23</td>
<td>24</td>
<td>25</td>
</tr>
<tr>
<td>26</td>
<td>27</td>
<td>28</td>
<td>29</td>
<td>30</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Labor Day
Here the $\phi$-scale lies the same distance $d$ above the paper as the $L$-scale lies below it but they are flattened to the paper. First, points $A$ and $B$ are joined by a line. Then for a given $L$ (point $C$), all four points will be coplanar if the point on the flattened $\phi$-scale is the same distance from $AB$ as $C$ and on a line parallel to $AB$. A transparent overlay of parallel lines is used to find $\phi$.

Angular correction for land surveys

\[
\sin \phi \cos \phi \tan \epsilon - \phi \tan \epsilon = 0
\]

where $\epsilon$ and $\phi$ are the land survey angles. For this problem, we have

\[
\begin{align*}
\lambda - \lambda' &= 25.2^\circ \\
\lambda + \lambda' &= 72.5^\circ \\
L &= 116^\circ \\
\phi &= 87.5^\circ
\end{align*}
\]

Great Circle Distance

Here the $\phi$-scale lies the same distance $d$ above the paper as the $L$-scale lies below it. Navigation and Surveying
In 1885, Charles Lallemand, director general of the geodetic measurement of altitudes throughout France, published a hexagonal chart for determining the compass course correction (the magnetic deviation) due to iron in the ship, *Le Triomphe* for any navigable location on Earth. It is a stunning piece of work, combining measured values of magnetic variation around the world with eight magnetic parameters of the ship also measured experimentally, all merged into a very complicated formula for magnetic deviation as seen at the top of the chart.

The sample calculation on the chart is described above:

1. The ship latitude and longitude is located among the curved lines on the leftmost map and a horizontal line is extended to the compass course $\zeta'$ in the center hourglass grid (a distance $Y_1 = B \sin \zeta'$ from the vertical green line).
2. The ship latitude and longitude is found in the upper (for a northerly heading) or lower (for a southerly heading) map and a line parallel to the grid is extended to the corresponding compass course in the twisted grid (a distance $Y_2 = A + C \cos \zeta' + D \sin 2 \zeta' + E \cos \zeta'$ from the angled green line).
3. A translucent hexagonal overlay (shown in blue) is overlaid so that two arms pass through the two marked points. Through a geometric exercise, it can be shown that the magnetic deviation (*compass correction*) $Y_3$ on the third scale is the sum of $Y_1$ and $Y_2$.

$\zeta'$ is the current ship compass reading (*or compass course*)

$H$ and $\theta$ are the horizontal component and dip angle of the Earth’s magnetic variation at the ship location

$A$, $D$, $E$, $\lambda$, $c$, $f$, $P$ and $Q$ are magnetic parameters measured for *Le Triomphe*
<table>
<thead>
<tr>
<th>Sunday</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>New Year's Day</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
<td>21</td>
<td>22</td>
</tr>
<tr>
<td>23</td>
<td>24</td>
<td>25</td>
<td>26</td>
<td>27</td>
<td>28</td>
<td>29</td>
</tr>
<tr>
<td>30</td>
<td>31</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Here the h₁ and h₂ scales are identical, and two ranges are marked on different sides of the scales.

To generalize these from u, v, w to f₁(u), f₂(v), f₃(w), we assign to the scales the values for which f₁(u), f₂(v), f₃(w) will provide the values we see here.

The 3 real roots of a cubic equation ax³ + bx² + cx + d = 0 sum to –b/a, so a plot of x³ marked with its x-values provides a single scale marked with its x-values.

With proper mathematical preparation, two or more scales can share a curve, or even share the same values on that curve.

The area of

Graphical Computing
AD 1844-1974

with proper mathematical preparation, two or more scales can share a curve, or even share the same values on that curve. The 3 real roots of a cubic equation ax³ + bx² + cx + d = 0 sum to –b/a, so a plot of x³ marked with its x-values provides a single scale marked with its x-values.

With proper mathematical preparation, two or more scales can share a curve, or even share the same values on that curve.
Lalanne’s Universal Calculator

In 1844, Leon LaLanne created the first log-log plot in history, his *Universal Calculator*. The product of x and y is found from their intersection with the 45° lines, squares at the 45° line from the origin, cubes from the steeper (Cub) line from the origin or its wraparound, and various engineering and chemical formulas of roots and powers at their lines. Following the line to the edge continues a calculation to additional terms.

Trigonometric functions are plotted along the sides for use (or use of their inverses) in calculations as well.

LaLanne envisioned copies of his *Universal Calculator* posted in public squares and business meeting places for popular use.
<table>
<thead>
<tr>
<th>Sunday</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Election Day</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>14</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>15</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>16</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>21</td>
<td>22</td>
<td>23</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>25</td>
<td>26</td>
<td>27</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>29</td>
<td>30</td>
<td></td>
<td>31</td>
</tr>
</tbody>
</table>

*Election Day
Veteran’s Day
Thanksgiving*
Introduction

In 1844, Leon Lalanne succeeded in linearizing the curves $y = x$ by plotting the first log-log plot in history, thereby creating his Universal Calculator, chock-full of lines for common engineering calculations and capable of graphically computing formulas in powers or roots of $x$ (or of trigonometric functions in $x$) with ease. The year 1844 is taken here as the start of the Age of Graphical Computing. Other graphical methods evolved, and ultimately the field of nomography was invented in 1880 by Maurice d'Ocagne, a breakthrough in graphical computing so radical that it dominated the field of graphical computing until the spread of computers and electronic calculators in the early 1970s.

The nomograms in the calendar are representative of some of the variety once in use for graphical computing, but in no way does it approach a significant survey of this rich field of study. Perhaps a 2011 calendar will consider other designs, additional information on how to use nomograms, and the realistic potential of nomograms today. Nomograms can be created with geometric relations, but more extraordinary ones are nearly always created using a method of determinants developed by d'Ocagne. Sometimes in this calendar you will see an equation adjacent to a nomogram, in which the determinant of a matrix is set equal to zero. When the determinant is expanded, you will see that the resulting equation matches the overall equation of the nomogram. If the determinant is in a form where no variable appears in more than one row and the last column is all 1's, then the first two elements in each row represent the (x,y) location of the scale point for values of the variable(s) in that row.

Nomograms can be created with geometric relations, but more extraordinary ones are nearly always created using a method of determinants developed by d'Ocagne. Sometimes in this calendar you will see an equation adjacent to a nomogram, in which the determinant of a matrix is set equal to zero. When the determinant is expanded, you will see that the resulting equation matches the overall equation of the nomogram. If the determinant is in a form where no variable appears in more than one row and the last column is all 1's, then the first two elements in each row represent the (x,y) location of the scale point for values of the variable(s) in that row.

This 2010 calendar predominantly treats the field of nomography and the amazing variety of nomograms that can be created from it. Nomograms can be created with geometric relations, but more extraordinary ones are nearly always created using a method of determinants developed by d'Ocagne. Sometimes in this calendar you will see an equation adjacent to a nomogram, in which the determinant of a matrix is set equal to zero. When the determinant is expanded, you will see that the resulting equation matches the overall equation of the nomogram. If the determinant is in a form where no variable appears in more than one row and the last column is all 1's, then the first two elements in each row represent the (x,y) location of the scale point for values of the variable(s) in that row.
An Assortment of Nomograms

Nomograms existed for a variety of vector and complex number calculations. A compass is used here instead of a straightedge. A projection transformation for greater accuracy at large $x$.

Nomogram for $\sinh(A + jB) = p + jq$:

- $A$ and $B$ are swapped.
- $p$ and $q$ are swapped.
- The $\theta$ scale could actually stop at 45. For angles beyond this, $\theta \rightarrow 90 - \theta$ and $x$ and $y$ are swapped.

Rybner, Hoelscher, Meyer.
Introduction

It is difficult for us today to grasp the drudgery of complex arithmetic calculations, or even repeated simpler calculations, in the past. This was especially true with repetitive computations that required tables of roots, logarithms and trigonometric functions in such fields as astronomy, navigation, surveying, and a wide variety of military and engineering applications.

Have you ever had to calculate the positions of astronomical objects? Orbital calculations relative to an observer on the Earth require derivations and time-consuming solutions of spherical trigonometric equations. And yet these kinds of calculations were accomplished by ancients such as Vitruvius and Ptolemy in the days prior to the advent of calculators or computers, or even trigonometry or algebra, using methods of Descriptive Geometry that are rarely taught today.

The Greeks folded (rabatted) the fundamental great circles onto the page and performed intricate geometrical constructions to map the Earth-Sun relative motion and incorporate local measurements into global maps and sophisticated sundials.

Astrolabes, quadrants and other volvelles and dials evolved to perform more complex computations in graphical form. In 1610-1614, Joost Bürgi and John Napier invented logarithms, and mathematicians and scientists such as Johann Kepler created tables of logarithms to aid in computation. William Oughtred and others developed the slide rule in the 1600s based on the properties of logarithms, and the slide rule continued its dominant role in non-graphical computation until the early 1970s. The slide rule provided the greatest versatility in computing the vast variety of equations, but it required multiple error-prone steps to provide solutions, effort that was not decreased even when solving one equation repetitively.

Meanwhile, on the graphical front Rene Descartes created the Cartesian coordinate system in the 17th century, and mathematicians over the next two centuries laid the foundation for applied numerical mathematics in large part on this field of analytical geometry. A two-dimensional graph provided fast solutions to an engineering precision for a single equation in two variables, and more complicated families of curves or so-called intersection charts extended the use of Cartesian graphs to one additional variable.
The Age of Graphical Computing

A.D. 1844

A.D. 1974

A.D. 2010

Time line
Graphical Computers are fascinating artifacts in the history of mathematics. They possess an intrinsic charm well beyond their practical use.

- As a calculating aid graphical computers can solve very complicated formulas with amazing ease.
- As a curiosity graphical computers manifest the beauty of mathematics in a highly visual, highly creative way.

Most of the nomograms herein were created with the PyNomo software package of Leif Roschier found at http://www.pynomo.org. The calendar pages are based on an InDesign template created by Juliana Halvorson at http://www.graphmaster.com/calendarinstructions/. All other content ©2010 Ron Doerfler. Contact: rondoerfler@myreckonings.com